

## Meson-baryon effective chiral Lagrangians to $\mathcal{O}(q^3)$

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**ABSTRACT:** We construct the complete and minimal  $\mathcal{O}(q^2)$  and  $\mathcal{O}(q^3)$  three-flavour Lorentz invariant chiral meson-baryon Lagrangians for the first time in the literature. We compare with previous three-flavour studies reducing the number of independent monomials and adding new ones that were missing.

**KEYWORDS:** QCD, NLO Computations, Chiral Lagrangians.

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## 1. Introduction

The extension of Chiral Perturbation Theory (CHPT) [1–3] to the one baryon sector is not straightforward, since, employing dimensional regularization, higher order loops contribute to lower order calculations. This is a consequence of the fact that the nucleon mass does not vanish in the chiral limit [4], therefore the correspondence between loop and chiral expansion is lost. This shortcoming was overcome within the formalism of heavy baryon CHPT [5, 6], where most of the higher order calculations in baryon CHPT have been performed (for reviews, see e.g. [7, 8]). More recently, it was realized that chiral power counting and loop expansion can be reconciled with a Lorentz invariant formulation of baryon CHPT employing the so called infrared regularization [9, 10]. In the literature have appeared many one loop calculations realized employing this scheme, especially in SU(2) baryon CHPT [11–15]. This method has also the advantage of correctly keeping the analytical properties of physical amplitudes, that in some cases are lost in heavy baryon CHPT in the low energy region. On the other hand, the chiral pion-nucleon SU(2) Lagrangian is completely known up to  $\mathcal{O}(q^4)$  [16], both the relativistic and the heavy baryon projected.

In baryon CHPT, the two flavour effective field theory is more developed than the three flavour one. Actually, a priori it is not clear whether the three flavour meson-baryon

system can be treated perturbatively due to the relatively large kaon mass. Furthermore in this sector one has also to face the presence of resonances close to or even below the pertinent thresholds, e.g. the  $\Lambda(1405)$ .<sup>1</sup> Most of the calculations in SU(3) baryon CHPT have been performed within the heavy baryon approximation [28–33]. In [34] the complete renormalization of the generating functional for Green functions of quark currents between one baryon states in three flavour heavy baryon CHPT is performed up to  $\mathcal{O}(q^3)$ . Some calculations have been already done within the infrared regularization scheme [35, 10, 36–39] or within the extended on-mass-shell renormalization scheme [40, 41].

An important aspect of this relative lack of development of SU(3) baryon CHPT is the unsatisfactory way the  $\mathcal{O}(q^2)$  and, particularly, the  $\mathcal{O}(q^3)$  meson-baryon Lorentz invariant chiral Lagrangians are given in the literature. The main purpose of this work consists in filling this gap. Since its publication, Krause’s work [42] has been employed as a standard reference for the effective Lorentz invariant chiral meson-baryon Lagrangian with three flavours up to  $\mathcal{O}(q^3)$ . However, the number of monomials appearing there can be further reduced, as shown below in section 5. Furthermore, the presentation of the monomials given in [42] can be certainly improved allowing for a much easier manipulation. At  $\mathcal{O}(q^2)$  part of the meson-baryon effective chiral Lagrangian is given without derivation in [35]. Again, we find that this Lagrangian can be further reduced and given in more compact form.

The content of the paper is organized as follows. In section 2 we present the building blocks that will be used in the construction of the effective meson-baryon Lagrangian and then we discuss their symmetry properties in section 3. In this section we also establish the conditions to be obeyed by the monomials written with the building blocks, in order to obtain a Lagrangian invariant under the strong interaction symmetries. All the general relations employed to reduce the number of independent monomials are listed in section 4. More specific manipulations are given in appendix A. Our final expressions for the  $\mathcal{O}(q^2)$  and  $\mathcal{O}(q^3)$  Lagrangians are displayed in section 5. Finally, in section 6 we summarize our main conclusions.

## 2. General framework and building blocks

The procedure for constructing non-linear effective chiral symmetric Lagrangians is standard [43]. We briefly sketch this procedure below.

QCD with three massless quarks,  $u$ ,  $d$ , and  $s$ , exhibits a global  $SU(3)_L \otimes SU(3)_R$  chiral symmetry, which is spontaneously broken to the subgroup  $SU(3)_V$ , with  $V = L + R$ . In order to write down the chiral invariant effective Lagrangian, it is convenient to promote the chiral symmetry to a local one introducing external hermitian  $3 \times 3$  matrix fields  $s(x)$ ,  $p(x)$ ,  $v_\mu(x)$  and  $a_\mu(x)$  which couple to scalar, pseudoscalar, vector and axial quark currents, respectively, as follows

$$\mathcal{L} = \mathcal{L}_{\text{QCD}}^0 + \bar{q}\gamma^\mu(v_\mu + \gamma_5 a_\mu)q - \bar{q}(s - i\gamma_5 p)q. \tag{2.1}$$

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<sup>1</sup>The implementation of non-perturbative resummation methods within the chiral expansion has allowed the successful use of chiral Lagrangians for the study of scattering and production processes in SU(3) baryon CHPT [17–27].

Here,  $\mathcal{L}_{\text{QCD}}^0$  is the QCD Lagrangian with massless  $u$ ,  $d$  and  $s$  quarks and current quark masses appear in the scalar source as  $s(x) = \mathcal{M} + \dots$ , where  $\mathcal{M} = \text{diag}(m_u, m_d, m_s)$  is a  $3 \times 3$  matrix collecting the light quark masses. For the construction of the  $\text{SU}(3)_L \otimes \text{SU}(3)_R$  chiral invariant Lagrangian we impose the constraints  $\langle a_\mu \rangle = \langle v_\mu \rangle = 0$ .<sup>2</sup> Electromagnetic interactions are introduced through the external vector field  $v_\mu = |e|QA_\mu$ , where  $Q = \text{diag}(2, -1, -1)/3$  is the quark electrical charge matrix and  $A_\mu$  the photon field — notice that  $\langle v_\mu \rangle = 0$  in this case.

The  $\text{SU}(3)$  effective chiral Lagrangian describing the interactions of the lightest pseudoscalar meson and baryon octets and external sources (photons, ...) is obtained by constructing the most general Lagrangian which is invariant under  $\text{SU}(3)_L \otimes \text{SU}(3)_R$  transformations and satisfies strong interaction symmetries.

The relevant degrees of freedom in the effective meson-baryon Lagrangian are the spontaneous chiral symmetry breaking Goldstone bosons and the octet of  $J^P = \frac{1}{2}^+$  baryons. Goldstone bosons are represented by a matrix field  $u(\Phi)$  which transforms under a general chiral rotation  $g = (g_L, g_R) \in \text{SU}(3)_L \otimes \text{SU}(3)_R$  as

$$u \longrightarrow u' = g_R u h^\dagger(g, u) = h(g, u) u g_L^\dagger \quad (2.2)$$

according to the standard non-linear realization [43], with  $h(g, u) \in \text{SU}(3)_V$ .

The octet of  $J^P = \frac{1}{2}^+$  baryons is arranged in a  $3 \times 3$  traceless matrix  $B$ ,

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix} \quad (2.3)$$

and corresponds to massive fields in the adjoint  $\text{SU}(3)_V$  representation transforming as

$$B \longrightarrow B' = h(g, u) B h^\dagger(g, u) \quad (2.4)$$

under chiral transformations [43].

The basic building blocks we use to construct the effective chiral Lagrangian are

$$\begin{aligned} u_\mu &= i\{u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu)u^\dagger\}, \\ \chi_\pm &= u^\dagger \chi u^\dagger \pm u \chi^\dagger u, \\ f_\pm^{\mu\nu} &= u F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu} u, \end{aligned} \quad (2.5)$$

where  $\chi = 2B_0(s + ip)$  and  $B_0 = -\langle 0|\bar{q}q|0\rangle/F^2$ , with  $\langle 0|\bar{q}q|0\rangle$  the  $\text{SU}(3)$  quark condensate and  $F$  the pion weak decay constant, both in the chiral limit. Here,

$$\begin{aligned} F_R^{\mu\nu} &= \partial^\mu r^\nu - \partial^\nu r^\mu - i[r^\mu, r^\nu], & r^\mu &= v^\mu + a^\mu, \\ F_L^{\mu\nu} &= \partial^\mu l^\nu - \partial^\nu l^\mu - i[l^\mu, l^\nu], & l^\mu &= v^\mu - a^\mu, \end{aligned} \quad (2.6)$$

are the external field strength tensors. The matrices  $u_\mu$ , and  $f_\pm^{\mu\nu}$  are traceless since we impose  $\langle v_\mu \rangle = \langle a_\mu \rangle = 0$ .

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<sup>2</sup>Here and in the rest of the paper,  $\langle X \rangle$  stands for the flavour trace of  $X$ .

The operators in (2.5) or any product thereof transform under  $SU(3)_L \otimes SU(3)_R$  transformations as  $X \rightarrow h X h^\dagger$  and their covariant derivative reads

$$D_\mu X = \partial_\mu X + [\Gamma_\mu, X], \tag{2.7}$$

where  $\Gamma_\mu$  is the chiral connection,

$$\Gamma_\mu = \frac{1}{2} \{ u^\dagger (\partial_\mu - i r_\mu) u + u (\partial_\mu - i l_\mu) u^\dagger \}. \tag{2.8}$$

We collectively call the operators in (2.5) and their covariant derivatives “chiral fields”.

For the construction of the effective Lagrangian the two relations

$$[D_\mu, D_\nu] X = \frac{1}{4} [[u_\mu, u_\nu], X] - \frac{i}{2} [f_{\mu\nu}^+, X], \tag{2.9}$$

$$D_\nu u_\mu - D_\mu u_\nu = f_{\mu\nu}^-, \tag{2.10}$$

turn out to be very useful. The first relation allows to consider only symmetric products of covariant derivatives while the second one to take just symmetrized covariant derivatives acting on  $u_\mu$ ,

$$h_{\mu\nu} = D_\mu u_\nu + D_\nu u_\mu. \tag{2.11}$$

### 3. Construction of allowed monomials

The chiral dimension of the building blocks in (2.5) is

$$\begin{aligned} u_\mu &\sim \mathcal{O}(q), \\ \chi_\pm, f_{\mu\nu}^\pm &\sim \mathcal{O}(q^2). \end{aligned} \tag{3.1}$$

The action of  $n$  covariant derivatives on any of the fields in (2.5) increases of  $n$  units the chiral order. We cannot extend this chiral counting rule to the field  $B$  as the covariant derivative, when applied to a baryon field, counts as a quantity of  $\mathcal{O}(q^0)$ , since the baryon mass does not vanish in the chiral limit. However, the combination  $(i \not{D} - M_0)B$ , where  $M_0$  is the octet baryon mass in the chiral limit, can be considered a small quantity [42] of the order of the soft momenta associated with pseudoscalar and external fields. Then we have the chiral counting rules

$$\begin{aligned} B, \bar{B}, D_\mu B &\sim \mathcal{O}(q^0), \\ (i \not{D} - M_0)B &\sim \mathcal{O}(q). \end{aligned} \tag{3.2}$$

The elements of the Clifford algebra basis have the following chiral dimensions

$$\begin{aligned} \mathbf{1}, \gamma_\mu, \gamma_5 \gamma_\mu, \sigma_{\mu\nu} &\sim \mathcal{O}(q^0), \\ \gamma_5 &\sim \mathcal{O}(q), \end{aligned} \tag{3.3}$$

as  $\gamma_5$  couples the small and the large components of the baryon spinor. We refer to the assignment of chiral dimensions in the baryonic sector given in eqs. (3.2) and (3.3) as the covariant chiral counting.

	P	C	h.c.	$\chi_{\text{dim}}$	$p$	$c$	$h$
$u_\mu$	$-P_\mu^\nu u_\nu$	$u_\mu^T$	$u_\mu$	1	1	0	0
$f_{\mu\nu}^+$	$P_\mu^\lambda P_\nu^\sigma f_{\lambda\sigma}^+$	$-f_{\mu\nu}^{+T}$	$f_{\mu\nu}^+$	2	0	1	0
$f_{\mu\nu}^-$	$-P_\mu^\lambda P_\nu^\sigma f_{\lambda\sigma}^-$	$f_{\mu\nu}^{-T}$	$f_{\mu\nu}^-$	2	1	0	0
$\chi_+$	$\chi_+$	$\chi_+^T$	$\chi_+$	2	0	0	0
$\chi_-$	$-\chi_-$	$\chi_-^T$	$-\chi_-$	2	1	0	1
$\vec{D}_\mu$	$P_\mu^\nu \vec{D}_\nu$	$\overleftarrow{D}_\mu^T$	$\overleftarrow{D}_\mu$	1	0	0	0

**Table 1:** Parity (P), charge conjugation (C) and hermitic conjugation (h.c.) transformation properties and chiral dimension of the building blocks and of their covariant derivative.  $P_\mu^\nu \equiv \text{diag}(+1, -1, -1, -1)$  is the matrix associated with the parity operator. See (3.5), (3.7) and (3.8) for the definition of  $p$ ,  $c$  and  $h$ .

	$\chi_{\text{dim}}$	$p$	$c$	$h$
$\mathbf{1}$	0	0	0	0
$\gamma_5$	1	1	0	1
$\gamma_\mu$	0	0	1	0
$\gamma_5 \gamma_\mu$	0	1	0	0
$\sigma_{\mu\nu}$	0	0	1	0

**Table 2:** Parity (P), charge conjugation (C) and hermitic conjugation (h.c.) transformation properties and chiral dimension of the Clifford algebra elements. See (3.5), (3.7) and (3.8) for the definition of  $p$ ,  $c$  and  $h$ .

The transformation properties under parity (P), charge conjugation (C) and hermitic conjugation (h.c.) of the building blocks in (2.5) can be found in table 1, while in table 2, we give the corresponding properties of the matrices  $\Gamma$  in (3.3) when appearing in the baryon bilinear  $\langle \bar{B} \Gamma B \rangle$ .

We start by writing down all possible chiral symmetric monomials fulfilling strong interaction symmetries, that is, which are invariant under Lorentz and parity transformations and charge and hermitic conjugation. A generic term is a bilinear in baryon fields and can contain more than one trace in flavour space. For every term in the Lagrangian being a Lorentz scalar, the space-time indices coming from chiral fields, covariant derivatives, Clifford algebra basis elements and tensors  $g_{\mu\nu}$  and/or pseudotensors  $\varepsilon_{\mu\nu\alpha\beta}$ , must be suitably contracted.

We first consider monomials composed by one trace and afterwards we discuss the case with two traces. Since matrix fields do not commute, we have to take into account all possible orderings. To this end and to have terms whose transformation properties under charge and hermitic conjugation are easily obtained, it is convenient to employ the form:

$$X = \langle \bar{B}(A_1, \dots, (A_n, \Theta D^m B) \dots) \rangle. \quad (3.4)$$

The fields  $A_1, A_2, \dots, A_n$  can be single chiral fields or a combination of (anti)commutators thereof and  $(A_i, A_j)$  denotes either the commutator,  $[A_i, A_j]$ , or the anticommutator,

$\{A_i, A_j\}$ , of  $A_i$  and  $A_j$ . The symbol  $\Theta$  indicates the product of an element of the Clifford algebra basis,  $\Gamma$ , times metric tensors and/or Levi-Civita pseudotensors while  $D^m$  is a set of  $m \geq 0$  covariant derivatives acting on  $B$  in a totally symmetrized way. In the previous equation, for the sake of simplicity in the notation, we have not shown explicitly the space-time indices attached to  $A_i$ ,  $\Theta$  and  $D^m$ .

The invariance of a candidate monomial  $X$  under  $P$  is easily checked taking into account the following transformation properties under parity

$$\begin{aligned} & \langle \bar{B}(A_1, \dots, (A_n, \Theta D^m B) \dots) \rangle^P \\ &= (-1)^{p_1 + \dots + p_n + p_\Gamma + n_\varepsilon} \langle \bar{B}(A_1, \dots, (A_n, \Theta D^m B) \dots) \rangle, \end{aligned} \tag{3.5}$$

where  $n_\varepsilon$  is the number of Levi-Civita pseudotensors present in (3.4) and the values of the exponents follow from tables 1 and 2. The subscript in  $p_\Gamma$  refers to the Clifford algebra matrix  $\Gamma$  contained in  $\Theta$ , as explained above. From (3.5), it follows that a candidate term can occur in  $\mathcal{L}_{MB}$  only if

$$(-1)^{p_1 + \dots + p_n + p_\Gamma + n_\varepsilon} = 1. \tag{3.6}$$

We next examine how  $X$  transforms under charge and hermitic conjugation. Here we essentially follow the lines of the analysis in ref. [42]. We first consider the case without covariant derivatives acting on the baryon fields. Under charge conjugation the monomial (3.4) transforms as

$$\begin{aligned} & \langle \bar{B}(A_1, \dots, (A_n, \Theta B) \dots) \rangle^C \\ &= (-1)^{c_1 + \dots + c_n + c_\Gamma} \langle \bar{B}(A_n, \dots, (A_1, \Theta B) \dots) \rangle, \end{aligned} \tag{3.7}$$

where  $c_i$  and  $c_\Gamma$  are determined from tables 1 and 2, respectively. Analogously, under hermitic conjugation, we have

$$\begin{aligned} & \langle \bar{B}(A_1, \dots, (A_n, \Theta B) \dots) \rangle^\dagger \\ &= (-1)^{h_1 + \dots + h_n + h_\Gamma} \langle \bar{B}(A_n, \dots, (A_1, \Theta B) \dots) \rangle, \end{aligned} \tag{3.8}$$

with  $h_i$  and  $h_\Gamma$  determined again from tables 1 and 2, respectively. Using the identities

$$\begin{aligned} [A, [C, B]] &= [C, [A, B]] + [[A, C], B] \\ [A, \{C, B\}] &= \{C, [A, B]\} + \{[A, C], B\} \\ \{A, \{C, B\}\} &= \{C, \{A, B\}\} + [[A, C], B], \end{aligned} \tag{3.9}$$

we can bring the terms in the r.h.s. of eqs. (3.7) and (3.8) to a form in which the operators  $A_i$  appear in the same order as in the original monomial, plus additional pieces:

$$\begin{aligned} & \langle \bar{B}(A_n, \dots, (A_2, (A_1, \Theta B) \dots)) \rangle \\ &= \langle \bar{B}(A_1, (A_2, \dots, (A_n, \Theta B) \dots)) \rangle + \langle \bar{B}(\tilde{A}_1, \dots, (\tilde{A}_2, (\tilde{A}_m, \Theta B) \dots)) \rangle, \end{aligned} \tag{3.10}$$

where  $\tilde{A}_i$  are (anti)commutators of the fields  $A_i$ , with  $m < n$ . To guarantee charge conjugation invariance of the effective interaction constructed from the monomial  $X$ , the combination  $(X + X^C)/2$  must be taken. From this consideration and eqs. (3.7) and (3.10), we conclude that a term  $X$  as defined in (3.4) will appear in  $\mathcal{L}_{MB}$  only if

$$(-1)^{c_1 + \dots + c_n + c_\Gamma} = 1. \tag{3.11}$$

Using (3.7) and (3.8), it is easy to show that charge conjugation symmetric terms are either hermitian or anti-hermitian.

We now consider the possibility that type (3.4) monomials contain  $m$  covariant derivatives acting on the baryon field  $B$ . In this case under charge conjugation the monomial  $X$  transforms as

$$X^C = (-1)^{c_1+\dots+c_n+c_\Gamma} \langle \bar{B} \overleftarrow{D}^m(A_n, \dots, (A_1, \Gamma B)) \rangle. \quad (3.12)$$

After performing an integration by parts and eliminating a total derivative, we can apply Leibniz rule and obtain a term with the covariant derivatives acting again on  $B$  together with a sum of terms in which additional covariant derivatives operate on the chiral fields. According to the chiral counting in the mesonic sector, the latter are, at least, of one order higher, so that we end up with

$$X^C = (-1)^{c_1+\dots+c_n+c_\Gamma+m} \langle \bar{B}(A_n, \dots, (A_1, \Theta D^m B)) \rangle + \text{h.o.}, \quad (3.13)$$

where h.o. denotes higher order terms with covariant derivatives acting on the chiral fields  $A_i$ . Up to the order considered, these higher orders contributions can be neglected and the monomial  $X$  will appear in  $\mathcal{L}_{MB}$  only if

$$(-1)^{c_1+\dots+c_n+c_\Gamma+m} = 1. \quad (3.14)$$

This condition also explains why the covariant derivative acting on the baryon field is considered odd under charge and hermitic conjugation. However, the resulting effective interaction  $(X + X^C)/2$ , where  $X^C$  is given in (3.13) by removing the higher order terms, is not always exactly invariant under charge conjugation, but only up to the considered chiral order. Importantly, in the effective meson-baryon chiral Lagrangian we choose to have terms that are exactly invariant under charge conjugation, i.e., to keep also the higher order contributions in (3.13), thus the exact  $X^C$  as given in eq. (3.12) is used. In this way, the amplitudes calculated with  $\mathcal{L}_{MB}$  will obey exact crossing symmetry under the exchange of meson fields. This is, of course, a fundamental property of physical amplitudes and is well worth keeping it exactly.

In the effective Lagrangian, there can also appear terms which are the products of two or more flavour traces. Explicitly, they can be either the product of one term of type (3.4) times flavour traces of chiral fields or monomials where the  $\bar{B}$  and  $B$  matrix fields are contained in two different flavour traces. Thus a general monomial can have one of the following forms:

$$X_1 = \langle \bar{B}(A_1, \dots, (A_j, \Theta D^m B) \dots) \rangle \langle (A_{j+1}, \dots, (A_{n-2}, A_{n-1}) \dots) A_n \rangle; \quad (3.15)$$

$$X_2 = \langle \bar{B}(A_1, \dots, (A_{j-1}, A_j) \dots) \rangle \langle (A_{j+1}, \dots, (A_{k-1}, A_k) \dots) \Theta D^m B \rangle \\ \times \langle (A_{k+1}, \dots, (A_{n-2}, A_{n-1}) \dots) A_n \rangle. \quad (3.16)$$

One can have more traces involving chiral fields than those explicitly shown above; in these cases the extension of the discussion below is straightforward.

For  $X_1$ -type terms, parity transformation, charge and hermitic conjugation properties can be studied analogously to the one flavour trace case and conditions (3.6) and (3.14)



must be satisfied for these terms too. For  $X_2$ -type terms, i.e., with  $B$  and  $\bar{B}$  in different traces, one obtains that condition (3.6) has to be satisfied for transformations under parity but condition (3.14) for transformations under charge conjugation changes. This is due to the fact that under charge conjugation the monomial transforms as

$$X_2^C = (-1)^{c_1+\dots+c_n+c_\Gamma+m} \langle \bar{B}(A_{j+1}, \dots, (A_{n-1}, A_n) \dots) \rangle \langle (A_1, \dots, (A_{j-1}, A_j) \dots) \Theta D^m B \rangle \times \langle (A_{k+1}, \dots, (A_{n-2}, A_{n-1}) \dots) A_n \rangle + \text{h.o.} \quad (3.17)$$

The monomial  $X_2$  can always appear in  $\mathcal{L}_{MB}$ , even if condition (3.14) is not satisfied since it is not possible, using the (anti)commutator identities (3.9), to reobtain the original term. As in the case with only one trace, we will take the combination  $(X_i + X_i^C)/2$  with exact  $X_i^C$ ,  $i = 1, 2$ . As in that case and both for  $X_1$  and  $X_2$ , it is easy to show that charge conjugation invariant terms are either hermitian or anti-hermitian.

#### 4. Construction of the effective chiral meson-baryon Lagrangian

In this section, we outline the method employed to get a minimal set of effective meson-baryon monomials up to  $\mathcal{O}(q^3)$ . Listing the terms satisfying the required symmetry conditions is a straightforward operation. In  $SU_L(3) \otimes SU_R(3)$ , at this order, with  $\langle a_\mu \rangle = \langle v_\mu \rangle = 0$ , we just need to consider monomials with one and two flavour traces. The procedure we use to obtain a complete list of allowed monomials is as follows. For a fixed element of the Clifford algebra basis (3.3) and number of flavour traces, we write down *all* possible monomials with the smallest number of covariant derivatives acting on the baryon field  $B$  that fulfill the symmetry requirements discussed in the previous section. The number of covariant derivatives acting on  $B$  is then gradually increased for the same Clifford algebra basis element and number of flavour traces. The procedure is over when the addition of more covariant derivatives acting on  $B$  does not yield new independent monomials due to the relations (4.4)–(4.10) given below.

Once a complete list of allowed monomials is obtained, the main task consists in finding out a minimal set of linearly independent interaction terms. In order to minimize the number of terms, we extensively employed several relations, like (2.9) and (2.10). A fundamental mean to eliminate redundant monomials in  $\mathcal{L}_{MB}$  is the use of the equations of motion (EOM) satisfied by mesons and baryons at lowest chiral order,  $\mathcal{O}(q^2)$  and  $\mathcal{O}(q)$ , respectively. The lowest order EOM satisfied by the pseudoscalar mesons is [3],

$$D_\mu u^\mu = \frac{i}{2} \tilde{\chi}_-, \quad (4.1)$$

where  $\tilde{\chi}_- = \chi_- - \frac{1}{3} \langle \chi_- \rangle$ . In the following, we consider  $\chi_-$  as an independent structure. The lowest order EOM satisfied by the baryon matrix field is

$$i\gamma^\mu D_\mu B - M_0 B + \frac{F}{2} \gamma^\mu \gamma_5 [u_\mu, B] + \frac{D}{2} \gamma^\mu \gamma_5 \left( \{u_\mu, B\} - \frac{1}{3} \langle \{u_\mu, B\} \rangle \right) = 0, \quad (4.2)$$

so that,  $i\gamma^\mu D_\mu B - M_0 B = \mathcal{O}(q)$ , as already reported in (3.2). The constants  $D$  and  $F$  are the axial-vector couplings.

Another important relation for reducing the  $\mathcal{O}(q^3)$  Lagrangian is

$$D^2 u_\mu = \frac{1}{4} [u_\beta, u_\mu] u^\beta - \frac{i}{2} f_{\beta\mu}^+ u^\beta + D^\beta f_{\beta\mu}^- + \frac{i}{2} D_\mu \tilde{\chi}_- . \quad (4.3)$$

This equation is readily obtained by taking the derivative of (2.10), using (2.9) and finally applying the pseudoscalar meson EOM (4.1). We will therefore not consider  $D^2 u_\mu$  as an independent structure.

We have also employed SU(3) Cayley-Hamilton relations for reducing the number of independent monomials keeping the maximum number of terms with one flavour trace.

Equations (2.9) and (4.2) allow to derive a set of relations containing different Clifford algebra elements and different number of covariant derivatives acting on the  $B$  matrix field, namely,

$$\langle \bar{B}(A_1, \dots, (A_n, \Gamma^{[\alpha]\beta} D_\beta D^m B) \dots) \rangle \tilde{\Theta} \simeq 0, \quad (4.4)$$

$$\langle \bar{B}(A_1, \dots, (A_n, D^\alpha D^m B) \dots) \rangle \tilde{\Theta} \simeq -i M_0 \langle \bar{B}(A_1, \dots, (A_n, \gamma^\alpha D^m B) \dots) \rangle \tilde{\Theta}, \quad (4.5)$$

$$\langle \bar{B}(A_1, \dots, (A_n, \gamma_5 D^\alpha D^m B) \dots) \rangle \tilde{\Theta} \simeq 0, \quad (4.6)$$

$$\langle \bar{B}(A_1, \dots, (A_n, \Gamma \gamma^\alpha D^\beta D^m B) \dots) \rangle \tilde{\Theta} \simeq \langle \bar{B}(A_1, \dots, (A_n, \Gamma \gamma^\beta D^\alpha D^m B) \dots) \rangle \tilde{\Theta}, \quad (4.7)$$

$$\begin{aligned} & \langle \bar{B}(A_1, \dots, (A_n, \Gamma \sigma^{\alpha\beta} D^\lambda D^m B) \dots) \rangle \tilde{\Theta} + \langle \bar{B}(A_1, \dots, (A_n, \Gamma \sigma^{\beta\lambda} D^\alpha D^m B) \dots) \rangle \tilde{\Theta} \\ & + \langle \bar{B}(A_1, \dots, (A_n, \Gamma \sigma^{\lambda\alpha} D^\beta D^m B) \dots) \rangle \tilde{\Theta} \simeq 0, \end{aligned} \quad (4.8)$$

$$\begin{aligned} & \varepsilon_{\alpha\beta\tau\rho} \left[ \langle \bar{B}(A_1, \dots, (A_n, \Gamma \sigma^{\alpha\beta} D^\lambda D^m B) \dots) \rangle \right. \\ & \left. + 2 \langle \bar{B}(A_1, \dots, (A_n, \Gamma \sigma^{\beta\lambda} D^\alpha D^m B) \dots) \rangle \right] \tilde{\Theta} \simeq 0, \end{aligned} \quad (4.9)$$

$$\varepsilon_{\alpha\beta\tau\rho} \langle \bar{B}(A_1, \dots, (A_n, \Gamma \sigma^{\alpha\beta} D^\tau D^m B) \dots) \rangle \tilde{\Theta} \simeq 0 \quad (4.10)$$

which will be extensively used to reduce the number of covariant derivatives acting on  $B$ . Here,  $\Gamma^{[\alpha]\beta}$  stands for a Clifford algebra basis element with either two Lorentz indices  $\alpha\beta$  or one index  $\beta$ . In these equations we have explicitly shown the elements of the Clifford algebra basis that appear and  $\Gamma$  is either  $\mathbf{1}$  or  $\gamma_5$ . The symbol  $\tilde{\Theta}$  refers to products of metric tensors and Levi-Civita pseudotensors, while “ $\simeq$ ” means equal up to terms of higher order or up to terms of the same order but with less covariant derivatives acting on the matrix field  $B$ . This definition of  $\simeq$  is sensible since those structures of the same order but with a lower number of covariant derivatives are already taken into account according to the procedure we follow for writing down the list of allowed monomials.

Relations analogous to (4.4)–(4.10) can also be obtained for the case with two flavour traces, because what matters in their derivation is the Dirac algebra and the action on  $B$  of covariant derivatives. Relations (4.9) and (4.10) are obtained from (4.8) after contracting it with the pseudotensor  $\varepsilon_{\alpha\beta\tau\rho}$ . Another interesting result that follows from (4.4) and (4.5) is that terms containing  $D_\mu D^\mu D^m B$  can be discarded.

Further reduction of monomials is reached by performing more specific manipulations –see appendix A for details. We finally arrive to a minimal set of linearly independent terms to  $\mathcal{O}(q^3)$  which we present in the next section.

## 5. The effective Lorentz invariant chiral meson-baryon Lagrangians to order $q^3$

### 5.1 The order $q^2$ Lorentz invariant effective chiral meson-baryon Lagrangian

Following the procedure detailed in the previous sections, we write down the relativistic effective meson-baryon chiral Lagrangian with three flavours at  $\mathcal{O}(q^2)$ ,

$$\begin{aligned}
 \mathcal{L}_{MB}^{(2)} = & b_D \langle \bar{B} \{ \chi_+, B \} \rangle + b_F \langle \bar{B} [ \chi_+, B ] \rangle + b_0 \langle \bar{B} B \rangle \langle \chi_+ \rangle + \\
 & b_1 \langle \bar{B} [ u^\mu, [ u_\mu, B ] ] \rangle + b_2 \langle \bar{B} \{ u^\mu, \{ u_\mu, B \} \} \rangle + \\
 & b_3 \langle \bar{B} \{ u^\mu, [ u_\mu, B ] \} \rangle + b_4 \langle \bar{B} B \rangle \langle u^\mu u_\mu \rangle + \\
 & ib_5 \left( \langle \bar{B} [ u^\mu, [ u^\nu, \gamma_\mu D_\nu B ] ] \rangle - \langle \bar{B} \overleftarrow{D}_\nu [ u^\nu, [ u^\mu, \gamma_\mu B ] ] \rangle \right) + \\
 & ib_6 \left( \langle \bar{B} [ u^\mu, \{ u^\nu, \gamma_\mu D_\nu B \} ] \rangle - \langle \bar{B} \overleftarrow{D}_\nu \{ u^\nu, [ u^\mu, \gamma_\mu B ] \} \rangle \right) + \\
 & ib_7 \left( \langle \bar{B} \{ u^\mu, \{ u^\nu, \gamma_\mu D_\nu B \} \} \rangle - \langle \bar{B} \overleftarrow{D}_\nu \{ u^\nu, \{ u^\mu, \gamma_\mu B \} \} \rangle \right) + \\
 & ib_8 \left( \langle \bar{B} \gamma_\mu D_\nu B \rangle - \langle \bar{B} \overleftarrow{D}_\nu \gamma_\mu B \rangle \right) \langle u^\mu u^\nu \rangle + id_1 \langle \bar{B} \{ [ u^\mu, u^\nu ], \sigma_{\mu\nu} B \} \rangle + \\
 & id_2 \langle \bar{B} [ [ u^\mu, u^\nu ], \sigma_{\mu\nu} B ] \rangle + id_3 \langle \bar{B} u^\mu \rangle \langle u^\nu \sigma_{\mu\nu} B \rangle + d_4 \langle \bar{B} \{ f_+^{\mu\nu}, \sigma_{\mu\nu} B \} \rangle + \\
 & d_5 \langle \bar{B} [ f_+^{\mu\nu}, \sigma_{\mu\nu} B ] \rangle .
 \end{aligned} \tag{5.1}$$

We compared this Lagrangian with that of ref. [42]. We found that 3 of the structures given in that paper<sup>3</sup> are redundant and can be expressed in terms of the others using Cayley-Hamilton equation and the relation (4.5). In ref. [35] part of the  $\mathcal{O}(q^2)$  Lagrangian is given, the one interesting for the authors' investigation, but the term with coefficient  $b_9$  is also redundant and using Cayley-Hamilton equation can be written in terms of the monomials proportional to  $b_5 - b_8$  in eq. (5.1) or in ref. [35].

The SU(2) version of  $\mathcal{L}_{MB}^{(2)}$  is obtained by reducing  $u$  in (2.2) to a SU(2) matrix, containing just the pion fields, and the matrix field  $B$  in (2.3) to a column vector  $\Psi$  collecting the proton and the neutron fields.<sup>4</sup> The external matrix fields  $s(x)$ ,  $p(x)$ ,  $v_\mu(x)$  and  $a_\mu(x)$  introduced in section 2 are also reduced to hermitian  $2 \times 2$  traceless matrices. In particular electromagnetic interactions are introduced through the external vector field  $v_\mu = |e|QA_\mu$ , where  $Q = \text{diag}(2, -1)/3$  is the quark electrical charge matrix and  $A_\mu$  the photon field. Notice that in this case  $\langle v_\mu \rangle \neq 0$  and flavour traces of  $f_{\mu\nu}^+$  can appear in the SU(2) Lagrangian. We fully agree with the  $\mathcal{O}(q^2)$  relativistic SU(2) meson-baryon Lagrangian given in [44].

### 5.2 The order $q^3$ effective chiral meson-baryon Lagrangian

The meson-baryon SU(3) chiral Lagrangian at  $\mathcal{O}(q^3)$  contains 84 terms that can be generally written as

$$\mathcal{L}_{MB}^{(3)} = \sum_{i=1}^{84} h_i O_i . \tag{5.2}$$

<sup>3</sup>Every structure usually involves several monomials in the reduced notation of ref. [42].

<sup>4</sup>Of course, we have now to employ Cayley-Hamilton relations for  $2 \times 2$  matrices.

The monomials  $O_i$  are shown in table 3, where we also display the vertex with the lowest number of particles to which each interaction term gives contribution.

$i$	$O_i$	Contributes to vertex
1	$i \left( \langle \bar{B} \gamma_\mu D_{\nu\rho} B [u^\mu, h^{\nu\rho}] \rangle + \langle \bar{B} \overleftarrow{D}_{\nu\rho} \gamma_\mu B [u^\mu, h^{\nu\rho}] \rangle \right)$	$M_1 B_1 \rightarrow M_2 B_2$
2	$i \left( \langle \bar{B} [u^\mu, h^{\nu\rho}] \gamma_\mu D_{\nu\rho} B \rangle + \langle \bar{B} \overleftarrow{D}_{\nu\rho} [u^\mu, h^{\nu\rho}] \gamma_\mu B \rangle \right)$	$M_1 B_1 \rightarrow M_2 B_2$
3	$i \left( \langle \bar{B} u^\mu \rangle \langle h^{\nu\rho} \gamma_\mu D_{\nu\rho} B \rangle - \langle \bar{B} \overleftarrow{D}_{\nu\rho} h^{\nu\rho} \rangle \langle u^\mu \gamma_\mu B \rangle \right)$	$M_1 B_1 \rightarrow M_2 B_2$
4	$i \langle \bar{B} [u_\mu, h^{\mu\nu}] \gamma_\nu B \rangle$	$M_1 B_1 \rightarrow M_2 B_2$
5	$i \langle \bar{B} \gamma_\nu B [u_\mu, h^{\mu\nu}] \rangle$	$M_1 B_1 \rightarrow M_2 B_2$
6	$i \left( \langle \bar{B} u_\mu \rangle \langle h^{\mu\nu} \gamma_\nu B \rangle - \langle \bar{B} h^{\mu\nu} \rangle \langle u_\mu \gamma_\nu B \rangle \right)$	$M_1 B_1 \rightarrow M_2 B_2$
7	$i \langle \bar{B} \sigma_{\mu\nu} D_\rho B \{u^\mu, h^{\nu\rho}\} \rangle - i \langle \bar{B} \overleftarrow{D}_\rho \sigma_{\mu\nu} B \{u^\mu, h^{\nu\rho}\} \rangle$	$M_1 B_1 \rightarrow M_2 B_2$
8	$i \langle \bar{B} \{u^\mu, h^{\nu\rho}\} \sigma_{\mu\nu} D_\rho B \rangle - i \langle \bar{B} \overleftarrow{D}_\rho \{u^\mu, h^{\nu\rho}\} \sigma_{\mu\nu} B \rangle$	$M_1 B_1 \rightarrow M_2 B_2$
9	$i \langle \bar{B} u^\mu \sigma_{\mu\nu} D_\rho B h^{\nu\rho} \rangle - i \langle \bar{B} \overleftarrow{D}_\rho u^\mu \sigma_{\mu\nu} B h^{\nu\rho} \rangle$	$M_1 B_1 \rightarrow M_2 B_2$
10	$i \langle \bar{B} h^{\nu\rho} \sigma_{\mu\nu} D_\rho B u^\mu \rangle - i \langle \bar{B} \overleftarrow{D}_\rho h^{\nu\rho} \sigma_{\mu\nu} B u^\mu \rangle$	$M_1 B_1 \rightarrow M_2 B_2$
11	$i \left( \langle \bar{B} \sigma_{\mu\nu} D_\rho B \rangle - \langle \bar{B} \overleftarrow{D}_\rho \sigma_{\mu\nu} B \rangle \right) \langle u^\mu h^{\nu\rho} \rangle$	$M_1 B_1 \rightarrow M_2 B_2$
12	$\langle \bar{B} \gamma_5 \gamma_\nu B \{u_\mu u^\mu, u^\nu\} \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
13	$\langle \bar{B} \gamma_5 \gamma_\nu B u_\mu u^\nu u^\mu \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
14	$\langle \bar{B} u_\mu \gamma_5 \gamma_\nu B \{u^\mu, u^\nu\} \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
15	$\langle \bar{B} u_\mu u^\mu \gamma_5 \gamma_\nu B u^\nu \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
16	$\langle \bar{B} \{u_\mu u^\mu, u^\nu\} \gamma_5 \gamma_\nu B \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
17	$\langle \bar{B} \{u^\mu, u^\nu\} \gamma_5 \gamma_\nu B u_\mu \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
18	$\langle \bar{B} u_\mu u^\nu u^\mu \gamma_5 \gamma_\nu B \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
19	$\langle \bar{B} u^\nu \gamma_5 \gamma_\nu B u_\mu u^\mu \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
20	$\langle \bar{B} \{u^\nu, \gamma_5 \gamma_\nu B\} \rangle \langle u_\mu u^\mu \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
21	$\langle \bar{B} [u^\nu, \gamma_5 \gamma_\nu B] \rangle \langle u_\mu u^\mu \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
22	$\langle \bar{B} \{u_\mu, \gamma_5 \gamma_\nu B\} \rangle \langle u^\mu u^\nu \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
23	$\langle \bar{B} [u_\mu, \gamma_5 \gamma_\nu B] \rangle \langle u^\mu u^\nu \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
24	$\langle \bar{B} \gamma_5 \gamma_\nu B \rangle \langle u_\mu u^\mu u^\nu \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
25	$\langle \bar{B} u_\mu \rangle \langle [u^\mu, u^\nu] \gamma_5 \gamma_\nu B \rangle - \langle \bar{B} [u^\mu, u^\nu] \rangle \langle u_\mu \gamma_5 \gamma_\nu B \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
26	$i \langle \bar{B} \gamma^\tau B \{u^\mu, u^\nu, u^\rho\} \rangle \varepsilon_{\mu\nu\rho\tau}$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
27	$i \langle \bar{B} \{[u^\mu, u^\nu], u^\rho\} \gamma^\tau B \rangle \varepsilon_{\mu\nu\rho\tau}$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
28	$i \langle \bar{B} [u^\mu, u^\nu] \gamma^\tau B u^\rho \rangle \varepsilon_{\mu\nu\rho\tau}$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
29	$i \langle \bar{B} u^\rho \gamma^\tau B [u^\mu, u^\nu] \rangle \varepsilon_{\mu\nu\rho\tau}$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
30	$i \langle \bar{B} \gamma^\tau B \rangle \langle [u^\mu, u^\nu] u^\rho \rangle \varepsilon_{\mu\nu\rho\tau}$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
31	$\langle \bar{B} \gamma_5 \gamma_\mu D_{\nu\rho} B u^\mu u^\nu u^\rho \rangle + \langle \bar{B} \overleftarrow{D}_{\nu\rho} \gamma_5 \gamma_\mu B u^\mu u^\nu u^\rho \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
32	$\langle \bar{B} u^\mu \gamma_5 \gamma_\mu D_{\nu\rho} B u^\nu u^\rho \rangle + \langle \bar{B} \overleftarrow{D}_{\nu\rho} u^\mu \gamma_5 \gamma_\mu B u^\nu u^\rho \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
33	$\langle \bar{B} u^\mu u^\nu \gamma_5 \gamma_\mu D_{\nu\rho} B u^\rho \rangle + \langle \bar{B} \overleftarrow{D}_{\nu\rho} u^\mu u^\nu \gamma_5 \gamma_\mu B u^\rho \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
34	$\langle \bar{B} u^\mu u^\nu u^\rho \gamma_5 \gamma_\mu D_{\nu\rho} B \rangle + \langle \bar{B} \overleftarrow{D}_{\nu\rho} u^\mu u^\nu u^\rho \gamma_5 \gamma_\mu B \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
35	$\left( \langle \bar{B} \{u^\mu, \gamma_5 \gamma_\mu D_{\nu\rho} B\} \rangle + \langle \bar{B} \overleftarrow{D}_{\nu\rho} \{u^\mu, \gamma_5 \gamma_\mu B\} \rangle \right) \langle u^\nu u^\rho \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
36	$\left( \langle \bar{B} [u^\mu, \gamma_5 \gamma_\mu D_{\nu\rho} B] \rangle + \langle \bar{B} \overleftarrow{D}_{\nu\rho} [u^\mu, \gamma_5 \gamma_\mu B] \rangle \right) \langle u^\nu u^\rho \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$

$i$	$O_i$	Contributes to vertex
37	$\left(\langle \bar{B} \gamma_5 \gamma_\mu D_\nu B \rangle + \langle \bar{B} \overleftarrow{D}_\nu \gamma_5 \gamma_\mu B \rangle\right) \langle u^\mu u^\nu u^\rho \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
38	$i \left( \langle \bar{B} u^\mu \sigma^{\lambda\tau} D_\rho B \{u^\nu, u^\rho\} \rangle - \langle \bar{B} \overleftarrow{D}_\rho u^\mu \sigma^{\lambda\tau} B \{u^\nu, u^\rho\} \rangle \right) \varepsilon_{\mu\nu\lambda\tau}$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
39	$i \left( \langle \bar{B} \{u^\mu, \sigma^{\lambda\tau} D_\rho B\} \rangle - \langle \bar{B} \overleftarrow{D}_\rho \{u^\mu, \sigma^{\lambda\tau} B\} \rangle \right) \langle u^\nu u^\rho \rangle \varepsilon_{\mu\nu\lambda\tau}$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
40	$i \left( \langle \bar{B} [u^\mu, \sigma^{\lambda\tau} D_\rho B] \rangle - \langle \bar{B} \overleftarrow{D}_\rho [u^\mu, \sigma^{\lambda\tau} B] \rangle \right) \langle u^\nu u^\rho \rangle \varepsilon_{\mu\nu\lambda\tau}$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
41	$i \left( \langle \bar{B} \sigma^{\lambda\tau} D_\rho B \rangle - \langle \bar{B} \overleftarrow{D}_\rho \sigma^{\lambda\tau} B \rangle \right) \langle u^\mu u^\nu u^\rho \rangle \varepsilon_{\mu\nu\lambda\tau}$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
42	$i \left( \langle \bar{B} u^\mu \rangle \langle [u^\nu, u^\rho] \sigma^{\lambda\tau} D_\rho B \rangle + \langle \bar{B} \overleftarrow{D}_\rho [u^\nu, u^\rho] \rangle \langle u^\mu \sigma^{\lambda\tau} B \rangle \right) \varepsilon_{\mu\nu\lambda\tau}$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
43	$i \left( \langle \bar{B} u^\mu \rangle \langle \{u^\nu, u^\rho\} \sigma^{\lambda\tau} D_\rho B \rangle - \langle \bar{B} \overleftarrow{D}_\rho \{u^\nu, u^\rho\} \rangle \langle u^\mu \sigma^{\lambda\tau} B \rangle \right) \varepsilon_{\mu\nu\lambda\tau}$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
44	$\langle \bar{B} u^\mu \gamma_5 \gamma_\mu B \chi_+ \rangle$	$B_1 \rightarrow M_1 B_2$
45	$\langle \bar{B} \chi_+ \gamma_5 \gamma_\mu B u^\mu \rangle$	$B_1 \rightarrow M_1 B_2$
46	$\langle \bar{B} u^\mu \gamma_5 \gamma_\mu B \rangle \langle \chi_+ \rangle$	$B_1 \rightarrow M_1 B_2$
47	$\langle \bar{B} \gamma_5 \gamma_\mu B u^\mu \rangle \langle \chi_+ \rangle$	$B_1 \rightarrow M_1 B_2$
48	$\langle \bar{B} \gamma_5 \gamma_\mu B \rangle \langle u^\mu \chi_+ \rangle$	$B_1 \rightarrow M_1 B_2$
49	$\langle \bar{B} \gamma_5 \gamma_\mu B \{u^\mu, \chi_+\} \rangle$	$B_1 \rightarrow M_1 B_2$
50	$\langle \bar{B} \{u^\mu, \chi_+\} \gamma_5 \gamma_\mu B \rangle$	$B_1 \rightarrow M_1 B_2$
51	$\langle \bar{B} \{ \chi_-, \gamma_5 B \} \rangle$	$B_1 \rightarrow M_1 B_2$
52	$\langle \bar{B} [ \chi_-, \gamma_5 B ] \rangle$	$B_1 \rightarrow M_1 B_2$
53	$\langle \bar{B} \gamma_5 B \rangle \langle \chi_- \rangle$	$B_1 \rightarrow M_1 B_2$
54	$\langle \bar{B} \gamma_\mu B [ \chi_-, u^\mu ] \rangle$	$B_1 M_1 \rightarrow M_2 B_2$
55	$\langle \bar{B} [ \chi_-, u^\mu ] \gamma_\mu B \rangle$	$B_1 M_1 \rightarrow M_2 B_2$
56	$\langle \bar{B} u^\mu \rangle \langle \chi_- \gamma_\mu B \rangle - \langle \bar{B} \chi_- \rangle \langle u^\mu \gamma_\mu B \rangle$	$B_1 M_1 \rightarrow M_2 B_2$
57	$\langle \bar{B} \{ D_\mu f_+^{\mu\nu}, \gamma_\nu B \} \rangle$	$B_1 \rightarrow \gamma B_2$
58	$\langle \bar{B} [ D_\mu f_+^{\mu\nu}, \gamma_\nu B ] \rangle$	$B_1 \rightarrow \gamma B_2$
59	$i \langle \bar{B} \gamma_5 \gamma_\nu B [ u_\mu, f_+^{\mu\nu} ] \rangle$	$\gamma B_1 \rightarrow M_2 B_2$
60	$i \langle \bar{B} [ u_\mu, f_+^{\mu\nu} ] \gamma_5 \gamma_\nu B \rangle$	$\gamma B_1 \rightarrow M_2 B_2$
61	$i \left( \langle \bar{B} u_\mu \rangle \langle f_+^{\mu\nu} \gamma_5 \gamma_\nu B \rangle - \langle \bar{B} f_+^{\mu\nu} \rangle \langle u_\mu \gamma_5 \gamma_\nu B \rangle \right)$	$\gamma B_1 \rightarrow M_2 B_2$
62	$\langle \bar{B} \gamma^\tau B \{ u^\mu, f_+^{\nu\rho} \} \rangle \varepsilon_{\mu\nu\rho\tau}$	$\gamma B_1 \rightarrow M_2 B_2$
63	$\langle \bar{B} \{ u^\mu, f_+^{\nu\rho} \} \gamma^\tau B \rangle \varepsilon_{\mu\nu\rho\tau}$	$\gamma B_1 \rightarrow M_2 B_2$
64	$\langle \bar{B} u^\mu \gamma^\tau B f_+^{\nu\rho} \rangle \varepsilon_{\mu\nu\rho\tau}$	$\gamma B_1 \rightarrow M_2 B_2$
65	$\langle \bar{B} f_+^{\nu\rho} \gamma^\tau B u^\mu \rangle \varepsilon_{\mu\nu\rho\tau}$	$\gamma B_1 \rightarrow M_2 B_2$
66	$\langle \bar{B} \gamma^\tau B \rangle \langle u^\mu f_+^{\nu\rho} \rangle \varepsilon_{\mu\nu\rho\tau}$	$\gamma B_1 \rightarrow M_2 B_2$
67	$\left( \langle \bar{B} [ u^\mu, f_+^{\nu\rho} ] \sigma^{\lambda\tau} D_\mu B \rangle - \langle \bar{B} \overleftarrow{D}_\mu [ u^\mu, f_+^{\nu\rho} ] \sigma^{\lambda\tau} B \rangle \right) \varepsilon_{\nu\rho\lambda\tau}$	$\gamma B_1 \rightarrow M_2 B_2$
68	$\left( \langle \bar{B} \sigma^{\lambda\tau} D_\mu B [ u^\mu, f_+^{\nu\rho} ] \rangle - \langle \bar{B} \overleftarrow{D}_\mu \sigma^{\lambda\tau} B [ u^\mu, f_+^{\nu\rho} ] \rangle \right) \varepsilon_{\nu\rho\lambda\tau}$	$\gamma B_1 \rightarrow M_2 B_2$
69	$\left( \langle \bar{B} u^\mu \rangle \langle f_+^{\nu\rho} \sigma^{\lambda\tau} D_\mu B \rangle + \langle \bar{B} \overleftarrow{D}_\mu f_+^{\nu\rho} \rangle \langle u^\mu \sigma^{\lambda\tau} B \rangle \right) \varepsilon_{\nu\rho\lambda\tau}$	$\gamma B_1 \rightarrow M_2 B_2$
70	$\langle \bar{B} \{ D_\mu f_-^{\mu\nu}, \gamma_5 \gamma_\nu B \} \rangle$	$\gamma B_1 \rightarrow M_2 B_2$
71	$\langle \bar{B} [ D_\mu f_-^{\mu\nu}, \gamma_5 \gamma_\nu B ] \rangle$	$\gamma B_1 \rightarrow M_2 B_2$
72	$\langle \bar{B} \gamma_5 \gamma^\tau B \{ u^\mu, f_-^{\nu\rho} \} \rangle \varepsilon_{\mu\nu\rho\tau}$	$\gamma B_1 \rightarrow M_2 M_3 B_2$
73	$\langle \bar{B} \{ u^\mu, f_-^{\nu\rho} \} \gamma_5 \gamma^\tau B \rangle \varepsilon_{\mu\nu\rho\tau}$	$\gamma B_1 \rightarrow M_2 M_3 B_2$
74	$\langle \bar{B} f_-^{\nu\rho} \gamma_5 \gamma^\tau B u^\mu \rangle \varepsilon_{\mu\nu\rho\tau}$	$\gamma B_1 \rightarrow M_2 M_3 B_2$

$i$	$O_i$	Contributes to vertex
75	$\langle \bar{B} u^\mu \gamma_5 \gamma^\tau B f_-^{\nu\rho} \rangle \varepsilon_{\mu\nu\rho\tau}$	$\gamma B_1 \rightarrow M_2 M_3 B_2$
76	$\langle \bar{B} \gamma_5 \gamma^\tau B \rangle \langle u^\mu f_-^{\nu\rho} \rangle \varepsilon_{\mu\nu\rho\tau}$	$\gamma B_1 \rightarrow M_2 M_3 B_2$
77	$i \langle \bar{B} [u_\mu, f_-^{\mu\nu}] \gamma_\nu B \rangle$	$\gamma B_1 \rightarrow M_2 M_3 B_2$
78	$i \langle \bar{B} \gamma_\nu B [u_\mu, f_-^{\mu\nu}] \rangle$	$\gamma B_1 \rightarrow M_2 M_3 B_2$
79	$i (\langle \bar{B} u_\mu \rangle \langle f_-^{\mu\nu} \gamma_\nu B \rangle - \langle \bar{B} f_-^{\mu\nu} \rangle \langle u_\mu \gamma_\nu B \rangle)$	$\gamma B_1 \rightarrow M_2 M_3 B_2$
80	$i \left( \langle \bar{B} \sigma_{\nu\rho} D_\mu B \{u^\mu, f_-^{\nu\rho}\} \rangle - \langle \bar{B} \overleftarrow{D}_\mu \sigma_{\nu\rho} B \{u^\mu, f_-^{\nu\rho}\} \rangle \right)$	$\gamma B_1 \rightarrow M_2 M_3 B_2$
81	$i \left( \langle \bar{B} \{u^\mu, f_-^{\nu\rho}\} \sigma_{\nu\rho} D_\mu B \rangle - \langle \bar{B} \overleftarrow{D}_\mu \{u^\mu, f_-^{\nu\rho}\} \sigma_{\nu\rho} B \rangle \right)$	$\gamma B_1 \rightarrow M_2 M_3 B_2$
82	$i \left( \langle \bar{B} u^\mu \sigma_{\nu\rho} D_\mu B f_-^{\nu\rho} \rangle - \langle \bar{B} \overleftarrow{D}_\mu u^\mu \sigma_{\nu\rho} B f_-^{\nu\rho} \rangle \right)$	$\gamma B_1 \rightarrow M_2 M_3 B_2$
83	$i \left( \langle \bar{B} f_-^{\nu\rho} \sigma_{\nu\rho} D_\mu B u^\mu \rangle - \langle \bar{B} \overleftarrow{D}_\mu f_-^{\nu\rho} \sigma_{\nu\rho} B u^\mu \rangle \right)$	$\gamma B_1 \rightarrow M_2 M_3 B_2$
84	$i \left( \langle \bar{B} \sigma_{\nu\rho} D_\mu B \rangle - \langle \bar{B} \overleftarrow{D}_\mu \sigma_{\nu\rho} B \rangle \right) \langle u^\mu f_-^{\nu\rho} \rangle$	$\gamma B_1 \rightarrow M_2 M_3 B_2$

Table 3: Minimal set of linearly independent monomials of the SU(3) chiral meson-baryon Lagrangian of  $\mathcal{O}(q^3)$ . On the third column we give the vertex with the minimal number of mesons and photons to which each term contributes.

The list of SU(3)  $\mathcal{O}(q^3)$  monomials presented in Krause's work [42] is neither complete nor minimal. We have checked that 22 out of the 60 structures given in this reference can be expressed as linear combination of those already given. This can be done by applying the meson EOM (4.1), Cayley-Hamilton equations and the relations (4.4)–(4.10). In addition, several monomials in table 3 are lacking in [42], namely, the ones from  $O_7$  to  $O_{10}$  and from  $O_{38}$  to  $O_{41}$ .

We would like to point out that the monomial  $O_{41}$ , being of  $\mathcal{O}(q^3)$  in the covariant counting of (3.2) and (3.3), actually starts contributing at  $\mathcal{O}(q^4)$  to meson-baryon amplitudes in a non-covariant chiral counting. To see this, notice that in a non-covariant counting, the  $\mathcal{O}(q^3)$  contributions from  $O_{41}$  are generated when the index  $\rho$  is temporal and  $\lambda$  and  $\tau$  are both spatial. Then in this case one has

$$i \left( \langle \bar{B} \sigma^{ij} D_0 B \rangle - \langle \bar{B} \overleftarrow{D}_0 \sigma^{ij} B \rangle \right) \langle u^\mu u^\nu u^0 \rangle \varepsilon_{\mu\nu ij} = 0. \quad (5.3)$$

We have also derived the SU(2) version of the  $\mathcal{L}_{MB}^{(3)}$  meson-baryon Lagrangian in the same way as we did for the  $\mathcal{O}(q^2)$  Lagrangian (5.1) and found a full agreement with the one obtained in [44].

## 6. Summary and conclusions

As already mentioned in the Introduction, in the literature can be found several one loop calculations performed in baryon CHPT employing parts of the  $\mathcal{O}(q^3)$  three flavour Lagrangian (5.2). However, in this work, we derived for the first time the complete  $\mathcal{O}(q^2)$  and  $\mathcal{O}(q^3)$  Lorentz invariant SU(3) effective meson-baryon chiral Lagrangians, eqs. (5.1) and (5.2), respectively. We both reduced the number of independent monomials given

in previous studies [42, 35] and identified missing terms [42]. There is perfect agreement between the SU(2) reduction of the  $\mathcal{O}(q^2)$  and  $\mathcal{O}(q^3)$  relativistic Lagrangians we obtained and those of ref. [44]. We also gave  $\mathcal{L}_{\text{MB}}^{(2)}$  and  $\mathcal{L}_{\text{MB}}^{(3)}$  in a way that it is exactly invariant under charge conjugation.

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## A. Elimination of monomials

In this appendix we show details on how we have further reduced the number of monomials by applying the relations (2.9) and (2.10) from right to left and then reintroducing covariant derivatives. Integrating by parts and neglecting total derivatives, one then applies the baryon EOM (4.2) and its hermitic conjugate, and checks whether such monomials are independent or a combination of other ones already considered. We have also employed (4.8) with  $\Gamma = \gamma_5$  as explained below.

In this way, by applying eq. (2.9), we can remove the following monomial

$$i\langle \bar{B}\{[u_\sigma, [u_\rho, u_\eta]], \sigma_{\alpha\beta} D^\sigma B\}\rangle \varepsilon^{\alpha\beta\rho\eta} . \quad (\text{A.1})$$

As an intermediate step in the elimination of this monomial, we used the identity

$$\begin{aligned} \sigma_{\alpha\beta} \varepsilon^{\alpha\beta\rho\eta} &= 2i\gamma_5 \sigma^{\rho\eta} \\ &= 2\gamma_5 (g^{\rho\eta} - \gamma^\rho \gamma^\eta). \end{aligned} \quad (\text{A.2})$$

This is employed in order to contract space-time indices of covariant derivatives acting on  $B$  or  $\bar{B}$  with those of  $g^{\rho\eta} = (\gamma^\rho \gamma^\eta + \gamma^\eta \gamma^\rho)/2$  and of  $\gamma^\rho \gamma^\eta$  in the second line of (A.2) and then apply the baryon EOM (4.2).

Employing the first line of (A.2), together with the cyclic relation (4.8) with  $\Gamma = \gamma_5$ , we can relate the two monomials,

$$\begin{aligned} \varepsilon_{\alpha\beta\sigma\rho} \langle \bar{B}\{[f_+^{\sigma\rho}, u^\nu], \sigma^{\alpha\beta} D_\nu B\}\rangle, \\ \varepsilon_{\alpha\beta\sigma\rho} \langle \bar{B}\{[f_+^{\sigma\nu}, u^\rho], \sigma^{\alpha\beta} D_\nu B\}\rangle \end{aligned} \quad (\text{A.3})$$

and express the latter in terms of the former, modulo terms of higher order or terms already considered with less covariant derivatives acting on  $B$ .

One can proceed in a similar way for the monomials involving two flavour traces,

$$\begin{aligned} & \varepsilon_{\alpha\beta\sigma\rho} \left( \langle \bar{B} \sigma^{\alpha\beta} u_\eta \rangle \langle f_+^{\sigma\rho} D^\eta B \rangle - \langle \bar{B} f_+^{\sigma\rho} \rangle \langle u_\eta \sigma^{\alpha\beta} D^\eta B \rangle \right), \\ & \varepsilon_{\alpha\beta\sigma\rho} \left( \langle \bar{B} \sigma^{\alpha\beta} u_\eta \rangle \langle f_+^{\eta\sigma} D^\rho B \rangle - \langle \bar{B} f_+^{\eta\sigma} \rangle \langle u_\eta \sigma^{\alpha\beta} D^\rho B \rangle \right) \end{aligned} \quad (\text{A.4})$$

and remove the second monomial in (A.4).

The elimination of

$$\begin{aligned} & \varepsilon_{\alpha\beta\sigma\rho} \langle \bar{B} \{ D^\nu f_-^{\sigma\rho}, \sigma^{\alpha\beta} D_\nu B \} \rangle, \\ & \text{and } \varepsilon_{\alpha\beta\sigma\rho} \langle \bar{B} \{ D^\rho f_-^{\nu\sigma}, \sigma^{\alpha\beta} D_\nu B \} \rangle, \end{aligned} \quad (\text{A.5})$$

is done in two steps. First, we write down the second monomial above in terms of the first one and others already considered by applying (A.2) and the cyclic relation (4.8), with  $\Gamma = \gamma_5$  as done for (A.3) and (A.4). Next, the first monomial is removed by employing from right to left (2.10), and then applying repeatedly the baryon EOM together with (2.9) and (4.6).

## B. Field transformations and use of EOM

In section 4 we employed baryon EOM as a mean to eliminate redundant structures in the construction of the  $\mathcal{O}(q^2)$  and  $\mathcal{O}(q^3)$  effective meson-baryon Lagrangians. Here we discuss the equivalence between using EOM and performing baryon field transformations in order to minimize the number of terms in such Lagrangians. In the mesonic sector this equivalence was demonstrated in refs. [45], while within SU(2) baryon CHPT this issue was addressed in ref. [44].

Suppose that we are dealing with the list of  $\mathcal{O}(q^2)$  meson-baryon monomials, in which appears an operator of the form

$$\mathcal{O} = i \left( \langle \bar{B} A \not{D} B \rangle - \langle \bar{B} \overleftarrow{D} A B \rangle \right) \tilde{\Theta}, \quad (\text{B.1})$$

where  $A$  is of  $\mathcal{O}(q^2)$  and can be either a single chiral field or a product or a (anti)commutator of chiral fields. For the sake of simplicity, we take  $(-1)^{c_A} = (-1)^{h_A} = 1$ . Our goal is getting rid of the term in eq. (B.1), which contains a structure present in the baryon EOM (4.2) and in its hermitic conjugate. To this end, we perform the following transformation on the baryon fields

$$\begin{aligned} B & \longrightarrow B' = (1 - A)B, \\ \bar{B} & \longrightarrow \bar{B}' = \bar{B}(1 - A), \end{aligned} \quad (\text{B.2})$$

which is actually a field translation. Let us consider the effect produced by this transformation in the  $\mathcal{O}(q)$  effective meson-baryon Lagrangian,

$$\mathcal{L}_{MB}^{(1)} = \langle \bar{B} (i\gamma^\mu D_\mu - M_0) B \rangle + \frac{D}{2} \langle \bar{B} \gamma_\mu \gamma_5 \{ u^\mu, B \} \rangle + \frac{F}{2} \langle \bar{B} \gamma_\mu \gamma_5 [ u^\mu, B ] \rangle. \quad (\text{B.3})$$



Inserting the new fields  $\bar{B}'$ ,  $B'$ , we obtain

$$\mathcal{L}_{MB}^{(1)} \longrightarrow \mathcal{L}_{MB}^{(1)} - i \left( \langle \bar{B} A \not{D} B \rangle - \langle \bar{B} \overleftarrow{D} A B \rangle \right) \tilde{\Theta} + 2M_0 \langle \bar{B} A B \rangle + \mathcal{O}(q^3). \quad (\text{B.4})$$

The second term in the r.h.s. exactly cancels the operator in eq. (B.1). This elimination corresponds to the relation (4.4) derived directly using the baryon EOM. The same procedure carried out at  $\mathcal{O}(q^2)$  can be repeated similarly at  $\mathcal{O}(q^3)$  and higher. Applying then Dirac algebra manipulations and finally the field translation (B.2), we can obtain the relations (4.4)-(4.10), which allow to eliminate monomials with covariant derivatives acting on the baryon fields in favor of terms with less covariant derivatives.

With the field transformation (B.2) we induce changes in higher order terms. However, since in an effective field theory we generate the list of all possible terms obeying the required symmetries, all these modifications only shift the values of some unknown coupling constants, but not the structure of the corresponding monomials.

The basic motivation for employing field transformations to minimize the number of terms in effective Lagrangians is the equivalence theorem. This theorem states that in renormalized field theories  $S$ -matrix elements (i.e. physical observables) are independent of the choice of the interpolating fields or, equivalently, are invariant under field transformations (provided the transformations satisfy certain properties) [46]. The equivalence theorem was extended to effective field theory in refs. [47].

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## Erratum

Recently, Frink and Meißner [48] pointed out that one can further reduce the number of monomials present in our  $\mathcal{O}(q^3)$  Lagrangian by six, passing from 84 to 78 in [48]. Here, we discuss also the findings of [48] since some of them are not accurate. Indeed, we find out that actually one can reduce by eight the number of independent monomials in our Lagrangian, but in addition, two monomials were wrongly discarded which, as a result, makes the agreement in the number of independent monomials with [48] complete.

Some Cayley-Hamilton relations involving monomials with five flavour matrices were missed by us, as correctly noticed in [48]. The technicalities of this point are explained in detail in the Appendix A of [48]. Along these lines, we find three Cayley-Hamilton relations among the monomials  $O_{12}$  to  $O_{25}$  of our Lagrangian that were not taken into account there. If these Cayley-Hamilton relations are used to discard only monomials involving the product of two flavour traces, then one monomial between  $O_{20}$ ,  $O_{22}$  and  $O_{24}$  and two more monomials between  $O_{21}$ ,  $O_{23}$  and  $O_{25}$  can be neglected. We choose to cast away  $O_{22}$ ,  $O_{23}$  and  $O_{25}$ . Thus, we agree with [48] that Cayley-Hamilton relations can be used to further reject three monomials from  $O_{12}$  to  $O_{25}$  in our Lagrangian. However, it is not possible to simultaneously disregard the monomials  $O_{20}$ ,  $O_{21}$  and  $O_{22}$  from that basis.

We find two other Cayley-Hamilton relations among the monomials  $O_{31}$  to  $O_{37}$  in our Lagrangian not considered before. They allow to discard two monomials between  $O_{35}$ ,  $O_{36}$  and  $O_{37}$ , as already remarked in [48]. We choose to cast aside  $O_{35}$  and  $O_{36}$ .

Another Cayley-Hamilton relation among the monomials  $O_{38}$  to  $O_{43}$  in our Lagrangian, not used in ref. [49], is found now. This fact is not commented in [48]. In this way, one can remove another monomial that we choose to be  $O_{43}$ .

In [49] we used a Cayley-Hamilton relation to cast away the one flavour trace monomial,

$$\widehat{O}_{35} = i \left( \langle \bar{B}\{u^\nu, u^\rho\}\sigma^{\lambda\tau} D_\rho B u^\mu \rangle - \langle \bar{B}\overleftarrow{D}_\rho\{u^\nu, u^\rho\}\sigma^{\lambda\tau} B u^\mu \rangle \right) \varepsilon_{\mu\nu\lambda\tau}, \quad (\text{C.1})$$

while all the other monomials neglected because of using the Cayley-Hamilton theorem contained more than one flavour trace. Here, due to large  $N_c$  counting, we prefer to neglect the two trace monomial  $O_{42}$  in [49] and put back  $\widehat{O}_{33}$  in our new basis for the  $\mathcal{O}(q^3)$  Lagrangian.

Apart from the missed Cayley-Hamilton relations in our Lagrangian, Frink and Meißner [48] also realized that only the symmetric combination of  $O_9$  and  $O_{10}$  in [49] is independent. Hence, only one of these two monomials should be considered and we keep  $O_9$ .

Frink and Meißner also noticed that the index ordering in the monomials  $O_{31}$ ,  $O_{33}$  and  $O_{34}$  in [49] do not match the conditions imposed by charge conjugation invariance. We want to point out that the difference between the index ordering in [49] and that which is exactly invariant under charge conjugation is  $\mathcal{O}(q^4)$ . However, we prefer monomials in the Lagrangian which are exactly charge conjugation invariant, because charge conjugation is a symmetry of strong interactions — see our comments in [49]. Then, we now take the ordering in the indices such that these monomials are exactly charge conjugation invariant.

As pointed out in [48] the relative sign between the two flavour traces in  $O_{41}$  should be plus instead of the minus in [49]. Once this is corrected  $O_{41}$  becomes of  $\mathcal{O}(q^4)$ . Then, the comment at the end of Section 5 of [49], though correct, is not relevant.

In addition, we notice that two independent monomials were wrongly discarded in [49]. These monomials are

$$\widehat{O}_{32} = \langle \bar{B} [[u_\mu, u_\nu], u^\rho] \gamma_5 \sigma^{\mu\nu} D_\rho B \rangle - \langle \bar{B} \overleftarrow{D}_\rho [[u_\mu, u_\nu], u^\rho] \gamma_5 \sigma^{\mu\nu} B \rangle \quad (\text{C.2})$$

and

$$\widehat{O}_{33} = \langle \bar{B} \gamma_5 \sigma^{\mu\nu} D_\rho B [[u_\mu, u_\nu], u^\rho] \rangle - \langle \bar{B} \overleftarrow{D}_\rho \gamma_5 \sigma^{\mu\nu} B [[u_\mu, u_\nu], u^\rho] \rangle. \quad (\text{C.3})$$

Summarizing the discussion above, we can take away from our  $\mathcal{O}(q^3)$  three-flavour meson-baryon Lagrangian the following eight monomials:  $O_{10}$ ,  $O_{22}$ ,  $O_{23}$ ,  $O_{25}$ ,  $O_{35}$ ,  $O_{36}$ ,  $O_{41}$  and  $O_{43}$ . In addition, we exchange  $O_{42}$  by  $\widehat{O}_{35}$  and add two monomials, namely,  $\widehat{O}_{32}$  and  $\widehat{O}_{33}$ , not included in [49]. We therefore end up with 78 independent monomials in the  $SU(3)$  meson-baryon chiral Lagrangian at  $\mathcal{O}(q^3)$  and agree fully with [48]. We give the complete list of the monomials present in the minimal  $SU(3)$  meson-baryon chiral invariant Lagrangian in table 4.

$$\mathcal{L}_{MB}^{(3)} = \sum_{i=1}^{78} h_i \widehat{O}_i. \quad (\text{C.4})$$

$i$	$\widehat{O}_i$	Contributes to vertex
1	$i \left( \langle \bar{B} \gamma_\mu D_{\nu\rho} B [u^\mu, h^{\nu\rho}] \rangle + \langle \bar{B} \overleftarrow{D}_{\nu\rho} \gamma_\mu B [u^\mu, h^{\nu\rho}] \rangle \right)$	$M_1 B_1 \rightarrow M_2 B_2$
2	$i \left( \langle \bar{B} [u^\mu, h^{\nu\rho}] \gamma_\mu D_{\nu\rho} B \rangle + \langle \bar{B} \overleftarrow{D}_{\nu\rho} [u^\mu, h^{\nu\rho}] \gamma_\mu B \rangle \right)$	$M_1 B_1 \rightarrow M_2 B_2$
3	$i \left( \langle \bar{B} u^\mu \rangle \langle h^{\nu\rho} \gamma_\mu D_{\nu\rho} B \rangle - \langle \bar{B} \overleftarrow{D}_{\nu\rho} h^{\nu\rho} \rangle \langle u^\mu \gamma_\mu B \rangle \right)$	$M_1 B_1 \rightarrow M_2 B_2$
4	$i \langle \bar{B} [u_\mu, h^{\mu\nu}] \gamma_\nu B \rangle$	$M_1 B_1 \rightarrow M_2 B_2$
5	$i \langle \bar{B} \gamma_\nu B [u_\mu, h^{\mu\nu}] \rangle$	$M_1 B_1 \rightarrow M_2 B_2$
6	$i \left( \langle \bar{B} u_\mu \rangle \langle h^{\mu\nu} \gamma_\nu B \rangle - \langle \bar{B} h^{\mu\nu} \rangle \langle u_\mu \gamma_\nu B \rangle \right)$	$M_1 B_1 \rightarrow M_2 B_2$
7	$i \langle \bar{B} \sigma_{\mu\nu} D_\rho B \{u^\mu, h^{\nu\rho}\} \rangle - i \langle \bar{B} \overleftarrow{D}_\rho \sigma_{\mu\nu} B \{u^\mu, h^{\nu\rho}\} \rangle$	$M_1 B_1 \rightarrow M_2 B_2$
8	$i \langle \bar{B} \{u^\mu, h^{\nu\rho}\} \sigma_{\mu\nu} D_\rho B \rangle - i \langle \bar{B} \overleftarrow{D}_\rho \{u^\mu, h^{\nu\rho}\} \sigma_{\mu\nu} B \rangle$	$M_1 B_1 \rightarrow M_2 B_2$
9	$i \langle \bar{B} u^\mu \sigma_{\mu\nu} D_\rho B h^{\nu\rho} \rangle - i \langle \bar{B} \overleftarrow{D}_\rho u^\mu \sigma_{\mu\nu} B h^{\nu\rho} \rangle$	$M_1 B_1 \rightarrow M_2 B_2$
10	$i \left( \langle \bar{B} \sigma_{\mu\nu} D_\rho B \rangle - \langle \bar{B} \overleftarrow{D}_\rho \sigma_{\mu\nu} B \rangle \right) \langle u^\mu h^{\nu\rho} \rangle$	$M_1 B_1 \rightarrow M_2 B_2$
11	$\langle \bar{B} \gamma_5 \gamma_\nu B \{u_\mu u^\mu, u^\nu\} \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
12	$\langle \bar{B} \gamma_5 \gamma_\nu B u_\mu u^\nu u^\mu \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
13	$\langle \bar{B} u_\mu \gamma_5 \gamma_\nu B \{u^\mu, u^\nu\} \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
14	$\langle \bar{B} u_\mu u^\mu \gamma_5 \gamma_\nu B u^\nu \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
15	$\langle \bar{B} \{u_\mu u^\mu, u^\nu\} \gamma_5 \gamma_\nu B \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
16	$\langle \bar{B} \{u^\mu, u^\nu\} \gamma_5 \gamma_\nu B u_\mu \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
17	$\langle \bar{B} u_\mu u^\nu u^\mu \gamma_5 \gamma_\nu B \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$

Table 4:

$i$	$\widehat{O}_i$	Contributes to vertex
18	$\langle \bar{B}u^\nu \gamma_5 \gamma_\nu B u_\mu u^\mu \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
19	$\langle \bar{B}\{u^\nu, \gamma_5 \gamma_\nu B\} \langle u_\mu u^\mu \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
20	$\langle \bar{B}[u^\nu, \gamma_5 \gamma_\nu B] \langle u_\mu u^\mu \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
21	$\langle \bar{B} \gamma_5 \gamma_\nu B \rangle \langle u_\mu u^\mu u^\nu \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
22	$i \langle \bar{B} \gamma^\tau B \{ [u^\mu, u^\nu], u^\rho \} \rangle \varepsilon_{\mu\nu\rho\tau}$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
23	$i \langle \bar{B} \{ [u^\mu, u^\nu], u^\rho \} \gamma^\tau B \rangle \varepsilon_{\mu\nu\rho\tau}$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
24	$i \langle \bar{B} [u^\mu, u^\nu] \gamma^\tau B u^\rho \rangle \varepsilon_{\mu\nu\rho\tau}$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
25	$i \langle \bar{B} u^\rho \gamma^\tau B [u^\mu, u^\nu] \rangle \varepsilon_{\mu\nu\rho\tau}$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
26	$i \langle \bar{B} \gamma^\tau B \rangle [u^\mu, u^\nu] u^\rho \rangle \varepsilon_{\mu\nu\rho\tau}$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
27	$\langle \bar{B} \gamma_5 \gamma_\mu D_{\nu\rho} B u^\mu u^\nu u^\rho \rangle + \langle \bar{B} \overleftarrow{D}_{\nu\rho} \gamma_5 \gamma_\mu B u^\rho u^\nu u^\mu \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
28	$\langle \bar{B} u^\mu \gamma_5 \gamma_\mu D_{\nu\rho} B u^\nu u^\rho \rangle + \langle \bar{B} \overleftarrow{D}_{\nu\rho} u^\mu \gamma_5 \gamma_\mu B u^\rho u^\nu \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
29	$\langle \bar{B} u^\mu u^\nu \gamma_5 \gamma_\mu D_{\nu\rho} B u^\rho \rangle + \langle \bar{B} \overleftarrow{D}_{\nu\rho} u^\nu u^\mu \gamma_5 \gamma_\mu B u^\rho \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
30	$\langle \bar{B} u^\mu u^\nu u^\rho \gamma_5 \gamma_\mu D_{\nu\rho} B \rangle + \langle \bar{B} \overleftarrow{D}_{\nu\rho} u^\rho u^\nu u^\mu \gamma_5 \gamma_\mu B \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
31	$\left( \langle \bar{B} \gamma_5 \gamma_\mu D_{\nu\rho} B \rangle + \langle \bar{B} \overleftarrow{D}_{\nu\rho} \gamma_5 \gamma_\mu B \rangle \right) \langle u^\mu u^\nu u^\rho \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
32	$\langle \bar{B} [[u_\mu, u_\nu], u^\rho] \gamma_5 \sigma^{\mu\nu} D_\rho B \rangle - \langle \bar{B} \overleftarrow{D}_\rho [[u_\mu, u_\nu], u^\rho] \gamma_5 \sigma^{\mu\nu} B \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
33	$\langle \bar{B} \gamma_5 \sigma^{\mu\nu} D_\rho B [[u_\mu, u_\nu], u^\rho] \rangle - \langle \bar{B} \overleftarrow{D}_\rho \gamma_5 \sigma^{\mu\nu} B [[u_\mu, u_\nu], u^\rho] \rangle$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
34	$i \left( \langle \bar{B} u^\mu \sigma^{\lambda\tau} D_\rho B \{ u^\nu, u^\rho \} \rangle - \langle \bar{B} \overleftarrow{D}_\rho u^\mu \sigma^{\lambda\tau} B \{ u^\nu, u^\rho \} \rangle \right) \varepsilon_{\mu\nu\lambda\tau}$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
35	$i \left( \langle \bar{B} \{ u^\nu, u^\rho \} \sigma^{\lambda\tau} D_\rho B u^\mu \rangle - \langle \bar{B} \overleftarrow{D}_\rho \{ u^\nu, u^\rho \} \sigma^{\lambda\tau} B u^\mu \rangle \right) \varepsilon_{\mu\nu\lambda\tau}$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
36	$i \left( \langle \bar{B} \{ u^\mu, \sigma^{\lambda\tau} D_\rho B \} \rangle - \langle \bar{B} \overleftarrow{D}_\rho \{ u^\mu, \sigma^{\lambda\tau} B \} \rangle \right) \langle u^\nu u^\rho \rangle \varepsilon_{\mu\nu\lambda\tau}$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
37	$i \left( \langle \bar{B} [u^\mu, \sigma^{\lambda\tau} D_\rho B] \rangle - \langle \bar{B} \overleftarrow{D}_\rho [u^\mu, \sigma^{\lambda\tau} B] \rangle \right) \langle u^\nu u^\rho \rangle \varepsilon_{\mu\nu\lambda\tau}$	$M_1 B_1 \rightarrow M_2 M_3 B_2$
38	$\langle \bar{B} u^\mu \gamma_5 \gamma_\mu B \chi_+ \rangle$	$B_1 \rightarrow M_1 B_2$
39	$\langle \bar{B} \chi_+ \gamma_5 \gamma_\mu B u^\mu \rangle$	$B_1 \rightarrow M_1 B_2$
40	$\langle \bar{B} u^\mu \gamma_5 \gamma_\mu B \rangle \langle \chi_+ \rangle$	$B_1 \rightarrow M_1 B_2$
41	$\langle \bar{B} \gamma_5 \gamma_\mu B u^\mu \rangle \langle \chi_+ \rangle$	$B_1 \rightarrow M_1 B_2$
42	$\langle \bar{B} \gamma_5 \gamma_\mu B \rangle \langle u^\mu \chi_+ \rangle$	$B_1 \rightarrow M_1 B_2$
43	$\langle \bar{B} \gamma_5 \gamma_\mu B \{ u^\mu, \chi_+ \} \rangle$	$B_1 \rightarrow M_1 B_2$
44	$\langle \bar{B} \{ u^\mu, \chi_+ \} \gamma_5 \gamma_\mu B \rangle$	$B_1 \rightarrow M_1 B_2$
45	$\langle \bar{B} \{ \chi_-, \gamma_5 B \} \rangle$	$B_1 \rightarrow M_1 B_2$
46	$\langle \bar{B} [\chi_-, \gamma_5 B] \rangle$	$B_1 \rightarrow M_1 B_2$
47	$\langle \bar{B} \gamma_5 B \rangle \langle \chi_- \rangle$	$B_1 \rightarrow M_1 B_2$
48	$\langle \bar{B} \gamma_\mu B [\chi_-, u^\mu] \rangle$	$M_1 B_1 \rightarrow M_2 B_2$
49	$\langle \bar{B} [\chi_-, u^\mu] \gamma_\mu B \rangle$	$M_1 B_1 \rightarrow M_2 B_2$
50	$\langle \bar{B} u^\mu \rangle \langle \chi_- \gamma_\mu B \rangle - \langle \bar{B} \chi_- \rangle \langle u^\mu \gamma_\mu B \rangle$	$M_1 B_1 \rightarrow M_2 B_2$
51	$\langle \bar{B} \{ D_\mu f_+^{\mu\nu}, \gamma_\nu B \} \rangle$	$B_1 \rightarrow \gamma B_2$
52	$\langle \bar{B} [D_\mu f_+^{\mu\nu}, \gamma_\nu B] \rangle$	$B_1 \rightarrow \gamma B_2$

Table 4:

$i$	$\widehat{O}_i$	Contributes to vertex
53	$i\langle\bar{B}\gamma_5\gamma_\nu B[u_\mu, f_+^{\mu\nu}]\rangle$	$\gamma B_1 \rightarrow M_2 B_2$
54	$i\langle\bar{B}[u_\mu, f_+^{\mu\nu}]\gamma_5\gamma_\nu B\rangle$	$\gamma B_1 \rightarrow M_2 B_2$
55	$i\left(\langle\bar{B}u_\mu\rangle\langle f_+^{\mu\nu}\gamma_5\gamma_\nu B\rangle - \langle\bar{B}f_+^{\mu\nu}\rangle\langle u_\mu\gamma_5\gamma_\nu B\rangle\right)$	$\gamma B_1 \rightarrow M_2 B_2$
56	$\langle\bar{B}\gamma^\tau B\{u^\mu, f_+^{\nu\rho}\}\rangle\varepsilon_{\mu\nu\rho\tau}$	$\gamma B_1 \rightarrow M_2 B_2$
57	$\langle\bar{B}\{u^\mu, f_+^{\nu\rho}\}\gamma^\tau B\rangle\varepsilon_{\mu\nu\rho\tau}$	$\gamma B_1 \rightarrow M_2 B_2$
58	$\langle\bar{B}u^\mu\gamma^\tau Bf_+^{\nu\rho}\rangle\varepsilon_{\mu\nu\rho\tau}$	$\gamma B_1 \rightarrow M_2 B_2$
59	$\langle\bar{B}f_+^{\nu\rho}\gamma^\tau B u^\mu\rangle\varepsilon_{\mu\nu\rho\tau}$	$\gamma B_1 \rightarrow M_2 B_2$
60	$\langle\bar{B}\gamma^\tau B\rangle\langle u^\mu f_+^{\nu\rho}\rangle\varepsilon_{\mu\nu\rho\tau}$	$\gamma B_1 \rightarrow M_2 B_2$
61	$\left(\langle\bar{B}[u^\mu, f_+^{\nu\rho}]\sigma^{\lambda\tau}D_\mu B\rangle - \langle\bar{B}\overleftarrow{D}_\mu[u^\mu, f_+^{\nu\rho}]\sigma^{\lambda\tau}B\rangle\right)\varepsilon_{\nu\rho\lambda\tau}$	$\gamma B_1 \rightarrow M_2 B_2$
62	$\left(\langle\bar{B}\sigma^{\lambda\tau}D_\mu B[u^\mu, f_+^{\nu\rho}]\rangle - \langle\bar{B}\overleftarrow{D}_\mu\sigma^{\lambda\tau}B[u^\mu, f_+^{\nu\rho}]\rangle\right)\varepsilon_{\nu\rho\lambda\tau}$	$\gamma B_1 \rightarrow M_2 B_2$
63	$\left(\langle\bar{B}u^\mu\rangle\langle f_+^{\nu\rho}\sigma^{\lambda\tau}D_\mu B\rangle + \langle\bar{B}\overleftarrow{D}_\mu f_+^{\nu\rho}\rangle\langle u^\mu\sigma^{\lambda\tau}B\rangle\right)\varepsilon_{\nu\rho\lambda\tau}$	$\gamma B_1 \rightarrow M_2 B_2$
64	$\langle\bar{B}\{D_\mu f_-^{\mu\nu}, \gamma_5\gamma_\nu B\}\rangle$	$\gamma B_1 \rightarrow M_2 B_2$
65	$\langle\bar{B}[D_\mu f_-^{\mu\nu}, \gamma_5\gamma_\nu B]\rangle$	$\gamma B_1 \rightarrow M_2 B_2$
66	$\langle\bar{B}\gamma_5\gamma^\tau B\{u^\mu, f_-^{\nu\rho}\}\rangle\varepsilon_{\mu\nu\rho\tau}$	$\gamma B_1 \rightarrow M_2 M_3 B_2$
67	$\langle\bar{B}\{u^\mu, f_-^{\nu\rho}\}\gamma_5\gamma^\tau B\rangle\varepsilon_{\mu\nu\rho\tau}$	$\gamma B_1 \rightarrow M_2 M_3 B_2$
68	$\langle\bar{B}f_-^{\nu\rho}\gamma_5\gamma^\tau B u^\mu\rangle\varepsilon_{\mu\nu\rho\tau}$	$\gamma B_1 \rightarrow M_2 M_3 B_2$
69	$\langle\bar{B}u^\mu\gamma_5\gamma^\tau B f_-^{\nu\rho}\rangle\varepsilon_{\mu\nu\rho\tau}$	$\gamma B_1 \rightarrow M_2 M_3 B_2$
70	$\langle\bar{B}\gamma_5\gamma^\tau B\rangle\langle u^\mu f_-^{\nu\rho}\rangle\varepsilon_{\mu\nu\rho\tau}$	$\gamma B_1 \rightarrow M_2 M_3 B_2$
71	$i\langle\bar{B}[u_\mu, f_-^{\mu\nu}]\gamma_\nu B\rangle$	$\gamma B_1 \rightarrow M_2 M_3 B_2$
72	$i\langle\bar{B}\gamma_\nu B[u_\mu, f_-^{\mu\nu}]\rangle$	$\gamma B_1 \rightarrow M_2 M_3 B_2$
73	$i\left(\langle\bar{B}u_\mu\rangle\langle f_-^{\mu\nu}\gamma_\nu B\rangle - \langle\bar{B}f_-^{\mu\nu}\rangle\langle u_\mu\gamma_\nu B\rangle\right)$	$\gamma B_1 \rightarrow M_2 M_3 B_2$
74	$i\left(\langle\bar{B}\sigma_{\nu\rho}D_\mu B\{u^\mu, f_-^{\nu\rho}\}\rangle - \langle\bar{B}\overleftarrow{D}_\mu\sigma_{\nu\rho}B\{u^\mu, f_-^{\nu\rho}\}\rangle\right)$	$\gamma B_1 \rightarrow M_2 M_3 B_2$
75	$i\left(\langle\bar{B}\{u^\mu, f_-^{\nu\rho}\}\sigma_{\nu\rho}D_\mu B\rangle - \langle\bar{B}\overleftarrow{D}_\mu\{u^\mu, f_-^{\nu\rho}\}\sigma_{\nu\rho}B\rangle\right)$	$\gamma B_1 \rightarrow M_2 M_3 B_2$
76	$i\left(\langle\bar{B}u^\mu\sigma_{\nu\rho}D_\mu B f_-^{\nu\rho}\rangle - \langle\bar{B}\overleftarrow{D}_\mu u^\mu\sigma_{\nu\rho}B f_-^{\nu\rho}\rangle\right)$	$\gamma B_1 \rightarrow M_2 M_3 B_2$
77	$i\left(\langle\bar{B}f_-^{\nu\rho}\sigma_{\nu\rho}D_\mu B u^\mu\rangle - \langle\bar{B}\overleftarrow{D}_\mu f_-^{\nu\rho}\sigma_{\nu\rho}B u^\mu\rangle\right)$	$\gamma B_1 \rightarrow M_2 M_3 B_2$
78	$i\left(\langle\bar{B}\sigma_{\nu\rho}D_\mu B\rangle - \langle\bar{B}\overleftarrow{D}_\mu\sigma_{\nu\rho}B\rangle\right)\langle u^\mu f_-^{\nu\rho}\rangle$	$\gamma B_1 \rightarrow M_2 M_3 B_2$

Table 4: Minimal set of linearly independent monomials of the  $SU(3)$  chiral meson-baryon Lagrangian of  $\mathcal{O}(q^3)$ . On the third column we give the vertex with the minimal number of mesons and photons to which each term contributes.

In the previous list, the symbol  $D_{\nu\rho} = D_\nu D_\rho + D_\rho D_\nu$ .

In addition, we notice that the monomial  $O_{40}^{(3)}$  of [48] is not exactly charge conjugation invariant since those terms involving two covariant derivatives acting on the mesonic fields  $u_\alpha$  are missed. These contributions, though  $\mathcal{O}(q^5)$ , are needed to guarantee exact charge conjugation invariance.



## Added references

- [48] M. Frink and U.-G. Meißner, *On the chiral effective meson-baryon Lagrangian at third order\**, *Eur. Phys. J. A* **29** (2006) 255.
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